Improved Design of Multihole Directional Couplers Using an Iterative Technique

ROBERT S. ELLIOTT, LIFE FELLOW, IEEE, AND YONG UK KIM, MEMBER, IEEE

Abstract —The widely used first-order polynomial representation of the frequency response of a multihole directional coupler is assumed. The roots of this polynomial are displaced iteratively until a desired response is achieved. One possible outcome is a Chebyshev response, but the method is capable of improving on that result if all portions of the passband are not equally important. Further improvement can be achieved if the directivity is made to ripple around a somewhat higher level. This causes a minor sacrifice in bandwidth. Examples are given.

I. Introduction

The FAR-FIELD pattern of an equispaced linear antenna array can be represented by the polynomial

$$f(w)=\sum_{n=0}^{N} I_n w^n=\prod_{n=1}^{N}(w-w_n) \qquad (1)$$

in which I_n is the relative current in the nth element ($I_N=1$) and $w=\exp(j\psi)$, with $\psi=(2\pi d/\lambda)\cos\theta$, where d is the element spacing, λ the wavelength, and θ the pointing direction. In 1946 Dolph [1] showed how (1) could be mated to a Chebyshev polynomial with the result that the pattern consisted of a main beam symmetrically surrounded by side lobes of an equal prescribed height. Dolph also demonstrated that the beam width of the main lobe was the minimum achievable.

Other workers soon realized that Dolph's technique could also be applied to the design of multisection transformers, since their frequency response (to first order) can be represented by a polynomial similar to (1) above, with $f(w)$ becoming the normalized reflection coefficient ρ and with $\psi=-2\beta l$, where βl is the electrical length of each transformer section. Collin [2] and Cohn [3] independently demonstrated the Chebyshev design of transformers in 1955.

Subsequently Levy [4] provided an analysis and synthesis procedure for multiaperture directional couplers. Once again, the frequency response, to first order, could be linked to a polynomial of the type shown in (1) above, with $f(w)$ functionally related to the directivity of the coupler. Dolph's technique could thus also be used in the design of an array of equispaced holes that couple two identical waveguiding structures.

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R. S. Elliott is with the Department of Electrical Engineering, University of California, Los Angeles, CA 90024.
Y. U. Kim is with TRW, One Space Park, Redondo Beach, CA 90278.
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Although the Chebyshev design is optimum in the sense of providing maximum bandwidth for a specified lower bound to the directivity of the coupler in the passband, it is not optimum unless all portions of the passband are equally important. The technique to be described in what follows addresses that problem.

Further, a Chebyshev design provides a directivity-versus-frequency response for a lossless directional coupler, consisting of $N+1$ equispaced holes, that exhibits N poles (frequencies at which the directivity is infinite) interspersed by $N-1$ lobes of common height $D_{\min}$. It is possible, for example, to raise $D_{\min}$ to a higher value $D'_{\min}$ by dropping the poles down to a value slightly above $D'_{\min}$, thus causing the response to ripple, without seriously affecting the bandwidth. This rippled response has the interesting feature that it can be achieved with more than one distribution of hole sizes, thereby permitting the designer to choose that solution which is easiest to realize physically. The iterative technique which follows demonstrates how such rippled responses can be achieved.

II. Background

Consider the four-port directional coupler suggested by Fig. 1. It consists of two identical waveguiding structures sharing a common wall in which $N+1$ equispaced holes have been cut. With a unit signal injected at port 1, to first order the output signals at the four ports are

$$\text{ports 1 and 4: } \sum_{n=0}^{N} b_n e^{-j2n\beta d} \qquad (2)$$

$$\text{port 2: }\left[1+\sum_{n=0}^{N} c_n\right]e^{-jN\beta d} \qquad (3)$$

$$\text{port 3: }\left[\sum_{n=0}^{N} c_n\right]e^{-jN\beta d}. \qquad (4)$$

The signals at ports 1 and 4 are phase referenced to the zeroth hole; those at ports 2 and 3 are phase referenced to the Nth hole. The phase constant of the propagating mode is β and the interhole spacing is d. The forward and backward scattering coefficients for the nth hole are respectively c_n and b_n.

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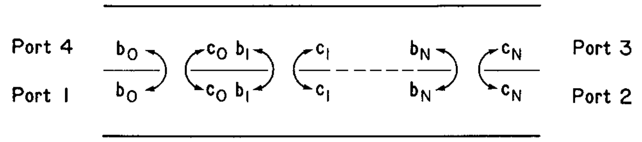

Fig. 1. A multihole directional coupler.

The coupling coefficient C is defined by the expression

$$C = 10\log_{10}\frac{P_1^{\text{in}}}{P_3^{\text{out}}} = 20\log_{10}\frac{1}{\left|\sum_{n=0}^{N} c_n\right|} \tag{5}$$

and the directivity D by

$$D = 10\log_{10}\frac{P_3^{\text{out}}}{P_4^{\text{out}}} = 20\log_{10}\frac{\left|\sum_{n=0}^{N} c_n\right|}{\left|\sum_{n=0}^{N} b_n e^{-j2n\beta d}\right|}. \tag{6}$$

We shall assume that either 1) the holes scatter symmetrically (example: they are in the common narrow wall between two identical rectangular waveguides), or 2) the holes scatter asymmetrically (examples: they are on the center line of the common broadwall between two identical rectangular waveguides or in the common ground plane between two identical striplines or microstrips). Thus $b_n = c_n$ or $b_n = -c_n$, and in either case

$$D = 20\log_{10}\frac{\left|\sum_{n=0}^{N} c_n\right|}{\left|\sum_{n=0}^{N} c_n e^{-j2n\beta d}\right|}. \tag{7}$$

In what follows we shall assume that over the frequency band of interest c_n is frequency independent and proceed to consider the following design problem: Given a desired level of coupling C, how does one keep the directivity D above a value $D_{\min}$ throughout the frequency band of interest?

It can be noted from (5) that, with C specified, $|\Sigma c_n|$ is known and therefore, from (7), that keeping D above $D_{\min}$ is equivalent to keeping $|\Sigma c_n \exp(-j2n\beta d)|$ below a related maximum value. Therefore let us employ the substitutions

$$\psi = -2\beta d \tag{8}$$

$$w = e^{j\psi} \tag{9}$$

and introduce the function

$$g(\beta d) = \sum_{n=0}^{N} c_n e^{-j2n\beta d} \tag{10}$$

so that

$$g(\psi) = \sum_{n=0}^{N} c_n e^{jn\psi} \tag{11}$$

$$g(w) = \sum_{n=0}^{N} c_n w^n = c_N \sum_{n=0}^{N} (c_n/c_N) w^n = c_N \prod_{n=1}^{N} (w - w_n). \tag{12}$$

In terms of the variable w we see from (7) that

$$D = 20\log_{10}\frac{|g(1)|}{|g(w)|} \tag{13}$$

whereas from (5) and (12)

$$|g(1)| = |c_N \sum_{n=0}^{N} (c_n/c_N)| = 10^{-C/20}. \tag{14}$$

A formula for $|g|_{\max}$ in the passband can be deduced from (13) and (14), viz.,

$$|g|_{\max} = |g(1)| \cdot 10^{-D_{\min}/20}. \tag{15}$$

Given C, (14) can be used to find $|g(1)|$ and then, given $D_{\min}$, (15) can be used to establish $|g|_{\max}$.

The design problem can now be viewed in the following light: Given (12) one needs to find the roots w_n so that $|g(w)| \leqslant |g|_{\max}$ in the desired band of frequencies. An iterative technique capable of finding the root positions for a wide variety of useful $D(\beta l)$ responses will be developed in the next two sections of this paper.

III. Analysis

From (12) it follows that

$$|g(w)/c_N|^2 = \prod_{n=1}^{N} (w - w_n)(w - w_n)^*. \tag{16}$$

Let the position of the nth root be denoted by

$$w_n = e^{a_n + jb_n}. \tag{17}$$

The insertion of (9) and (17) in (16) yields

$$|g(\psi)/c_N|^2 = \prod_{n=1}^{N} \left[1 - 2e^{a_n}\cos(\psi - b_n) + e^{2a_n}\right] \tag{18}$$

with ψ playing the role of surrogate for the frequency ν because of (8) and the fact that β is a function of ν.

We shall need to distinguish two cases: a) $N = 2M$ is an even number; b) $N = 2M+1$ is an odd number. Because of the physical requirement that all the coupling coefficients c_n have a common phase, the roots w_n must occur in complex conjugate pairs or be single roots on the real axis. Thus for $N = 2M$ (18) becomes

$$|g(\psi)/c_N|^2 = \prod_{n=1}^{M} \left[1 - 2e^{a_n}\cos(\psi - b_n) + e^{2a_n}\right] \cdot \left[1 - 2e^{a_n}\cos(\psi + b_n) + e^{2a_n}\right] \tag{19}$$

whereas, for $N = 2M+1$, (18) adopts the form

$$|g(\psi)/c_N|^2 = \left(1 + 2e^{a_0}\cos\psi + e^{2a_0}\right) \cdot \prod_{n=1}^{M} \left[1 - 2e^{a_n}\cos(\psi - b_n) + e^{2a_n}\right] \cdot \left[1 - 2e^{a_n}\cos(\psi + b_n) + e^{2a_n}\right]. \tag{20}$$

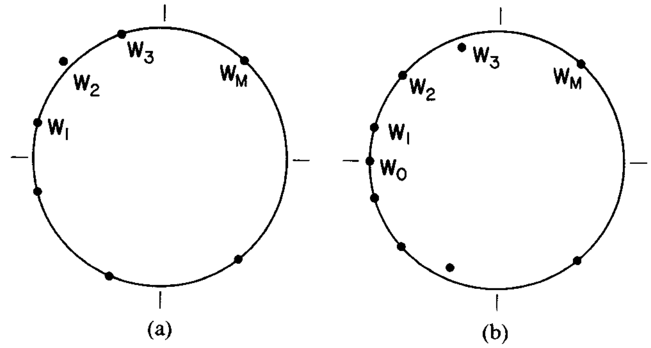

Fig. 2. Root indexing for odd and even numbers of holes. (a) $N+1=2M+1$. (b) $N+1=2M+2$.

In (20) the single root has been placed on the negative real axis by making $b_0=\pi$. The reason for this placement will become apparent shortly.

Possible root distributions for these two cases are shown in the w plane plots of Fig. 2. The unit circle $w=e^{j\psi}$ is displayed in both panels and some of the roots are placed on the unit circle, others being displaced radially inward or outward.

As the frequency changes and βd ranges from 0 to π, ψ varies from 0 to -2π and w makes one complete excursion clockwise around the unit circle. For $\beta d>\pi$ the pattern of response begins to repeat so only the range $0\leqslant\beta d\leqslant\pi$ need be considered. If all the roots w_n are placed on the unit circle, as w makes its excursion, whenever w coincides with a root, one of the factors in (12) is zero and the directivity becomes infinite. When w is approximately halfway between successive roots there is a minimum in the directivity. However, if some of the roots are off the unit circle, when w passes by such a root there is a finite peak in the directivity rather than a pole. Proper root placement can be seen to provide a variety of useful responses.

When doing analysis one assumes a set of root positions. As examples, if all the roots are placed at $-1+j0$ a maximally flat response for $D(\nu)$ results, whereas if the roots are equispaced an amount $2\pi/(N+1)$ along the unit circle, with a root missing at $1+j0$, all the coupling coefficients c_n are the same.

But in what follows we shall be interested in synthesis, where the desired response $D(\nu)$ is specified and the problem is to find the optimum placement of the roots w_n.

IV. Design Procedure

From (13) and (20) one can perceive that, for $N=2M+1$, i.e., for $2M+2$ holes,

$$D(\psi)=K_1-\left\{10\sum_{n=1}^{M}\log_{10}\left[1-2e^{a_n}\cos(\psi-b_n)+e^{2a_n}\right]+10\sum_{n=1}^{M}\log_{10}\left[1-2e^{a_n}\cos(\psi+b_n)+e^{2a_n}\right]+10\log_{10}\left[1+2e^{a_0}\cos\psi+e^{2a_0}\right]\right\} \tag{21}$$

with $K_1=20\log_{10}|g(0°)/c_N|$. For $N=2M$ one need only delete the ultimate term on the right side of (21).

When N, K_1, and a set of starting values a_n, b_n are given, $D(\psi)$ becomes a *known* function and can be compared to a *specified* function $S(\psi)$. If one forms the difference function

$$D(a_0,a_1,\cdots,a_M,b_1,b_2,\cdots,b_M,K_1,\psi)-S(\psi) \tag{22}$$

and holds ψ fixed at some value ψ_m while permitting the a_n's and b_n's to change slightly (K_1 must also change slightly to maintain $D=0$ at $\psi=0$), the total differential is

$$\delta(D-S)=\sum_{n=0}^{M}\left.\frac{\partial D}{\partial a_n}\right|_{P_m}\delta a_n+\sum_{n=1}^{M}\left.\frac{\partial D}{\partial b_n}\right|_{P_m}\delta b_n+\delta K_1. \tag{23}$$

The partial derivatives appearing in (23) can be deduced analytically from (21) and evaluated at the point

$$P_m(a_0,a_1,\cdots,a_M,b_1,b_2,\cdots,b_M,K_1,\psi_m).$$

The left side of (23) should be the *negative* of the difference, at ψ_m, between what is given and what is desired, both known quantities, since one wishes to find the changes δa_n, δb_n, and δK_1 that will eliminate this difference. One needs to select ψ_m carefully. For a case in which all the roots w_n are to be on the unit circle, the angles at which the peaks of the $M+1$ lobes in the D function occur in $-\pi\leqslant\psi\leqslant 0$ should be chosen. If $2P$ or $2P+1$ roots are to be off the unit circle, the angles at which the P or $P+1$ dips in the D function occur in $-\pi\leqslant\psi\leqslant 0$ should also be chosen. These sets comprise the ψ_m values at which one has all the relevant information about $D-S$.

When each of the ψ_m values just enumerated is used successively in (23), one obtains a deterministic set of simultaneous linear equations. Matrix inversion yields values for δa_n, δb_n, and δK_1. A new D function can be created by inserting the replacements $a_0+\delta a_0$, $a_n+\delta a_n$, $b_n+\delta b_n$, and $K_1+\delta K_1$ in (21). This new D function can be compared to $S(\psi)$ and if the agreement is not yet satisfactory the entire procedure can be repeated. Experience has shown in a variety of practical examples that normally only a few iterations are needed and that the choice of starting parameters in not critical.

With $D(\psi)$ satisfactorily close to $S(\psi)$ one can use the final root positions w_n to expand the product of factors $(w-w_n)$, thereby creating the polynomial whose coefficients are c_n/c_N. (cf. (12)). Then (14) can be used to determine c_N. With all the coupling coefficients known, the relation between coupling coefficient and hole size (obtained either experimentally or theoretically for the specific application) can be utilized to complete the design.

V. Illustrations of the Design Procedure

A Chebyshev response can be obtained readily using the procedure just described. For example, suppose one wishes to design a seven-hole coupler with $C=15$ dB and $D_{\min}=30$ dB. One can start with all six roots on the unit circle,

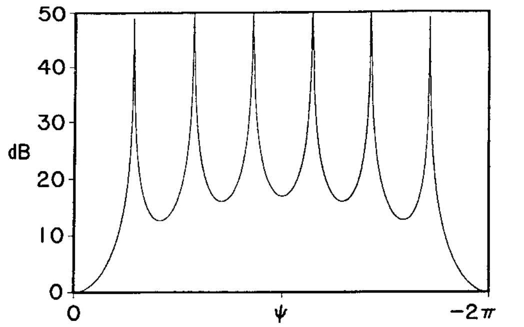

Fig. 3. Starting $D(\psi)$ function for seven-hole directional coupler

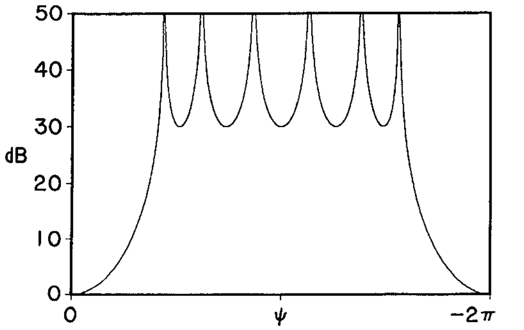

Fig. 4. Chebyshev response for seven-hole directional coupler. $D_{min} =$ 30 dB.

spaced $2\pi/7$ radians apart, but with a root missing at $w = 1 + j0$. This gives equal scattering coefficients c_n (all holes the same size) and, with $a_n = 0$ for all n, (21) becomes in this case

$$D(\psi) = K_1 - 10 \sum_{n=1}^{3} \{ \log_{10}[2 - 2\cos(\psi - b_n)] + \log_{10}[2 - 2\cos(\psi + b_n)] \} \quad (24)$$

with $b_1 = 6\pi/7$, $b_2 = 4\pi/7$, and $b_3 = 2\pi/7$. Since $D(0°) = 0$, one finds from (24) that $K_1 = -16.90$ dB. A plot of this starting $D(\psi)$ function is shown in Fig. 3. The inverted lobes which appear in this figure have minima which occur at $\psi = 0°$, $-74.08°$, $-127.42°$, $-180°$, $-232.58°$, $-285.92°$, and $-360°$. We find, using (24), that $D(0°) = 0$, $D(-74.08°) = 12.65$, $D(-127.42°) = 15.98$, and $D(-180°) = 16.90$ dB; the response is mirror symmetric around $\psi = -180°$. What is desired is that these inverted lobe minima all be at 30 dB.

Since all the roots are to remain on the unit circle, (23) simplifies for this case to involve just four unknowns: δb_1, δb_2, δb_3, and δK_1. Evaluation of $\partial D/\partial b_n$ at the ψ values $0°$, $-74.08°$, $-127.42°$, and $-180°$ permits construction of a 4 by 4 matrix, and inversion yields values for the unknowns and a new starting function $D(\psi)$. Two iterations bring all lobe minima within 0.1 dB of the specified value $D_{min} = 30$ dB, at which stage $b_1 = 2.81$, $b_2 = 1.96$, and $b_3 = 1.39$. When these values are placed in (21), calculations produce Fig. 4, which is seen to give the desired Chebyshev response. With the roots w_n known, multiplication of the factors $(w - w_n)$ in (12) permits identification of the relative scattering coefficients c_n/c_6, after which (14) yields the value of $|c_6|$. Table I lists the magnitudes of all seven scattering coefficients for this case.

TABLE I
MAGNITUDES OF THE SCATTERING COEFFICIENTS FOR A SEVEN-HOLE DIRECTIONAL COUPLER CHEBYSHEV DESIGN (FIG. 4) AND MODIFIED CHEBYSHEV DESIGN (FIG. 5)

n	0	1	2	3	4	5	6
$\|c_n\|$ for Figure 4	.0106	.0229	.0352	.0403	.0352	.0229	.0106
$\|c_n\|$ for Figure 5	.0101	.0232	.0351	.0410	.0351	.0232	.0101

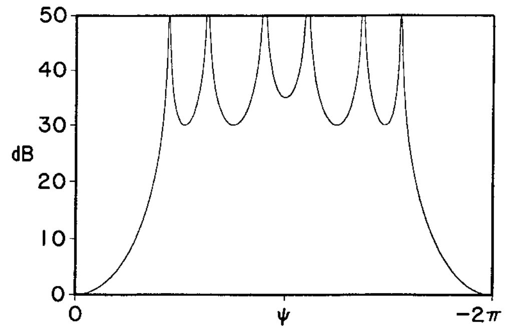

Fig. 5. Modified Chebyshev response for seven-hole directional coupler.

Of course, this design could have been achieved by conventional means [4]. However, consider the situation in which the central portion of the frequency band of the directional coupler is more important. Suppose, for example, that the response in Fig. 4 is to be modified so that the central inverted lobe reaches down only to 35 dB. Once again, Fig. 3 can be used as the starting $D(\psi)$ function[1] and the procedure just described can be followed, the only difference being that $S(-180°) = 35$ dB instead of 30 dB. One finds that two iterations bring all lobe minima within 0.1 dB of specification. The final root positions are $b_1 = 2.72$, $b_2 = 1.94$, and $b_3 = 1.37$. With these values known, (12) and (14) can once again be used to deduce the magnitudes of the scattering coefficients. Their values are entered in Table I. By comparing these entries with those for the Chebyshev 30 dB design, one can see that the physical realizability appears to be no more difficult. However, precise fabrication is needed to differentiate the two designs. The type of response shown in Fig. 5, where the lobe minima are not all the same, *cannot* be achieved by the conventional Chebyshev procedure.

A more important application of this iterative technique involves moving roots off the unit circle. Consider, for example, the case of a six-hole directional coupler designed so that the directivity ripples ± 1 dB around $D_{av} = 34$ dB. This ensures that $D_{min} = 33$ dB, which is 3 dB better than the 30 dB Chebyshev design shown in Fig. 6. That design was achieved using the iterative technique in exactly the manner that produced Fig. 4 and gives the root positions $b_0 = \pi$, $b_1 = 2.25$, and $b_2 = 1.60$ rad. We can create a starting pattern by arbitrarily choosing $a_0 = a_1 = a_2 = 0.1$,

[1]One could use Fig. 4 as the starting $D(\psi)$ if it had already been constructed.

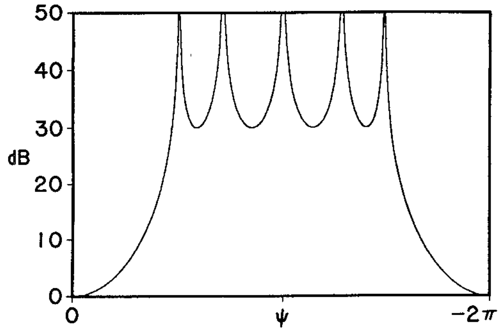

Fig. 6. Chebyshev response for six-hole directional coupler. $D_{\min} = 30$ dB.

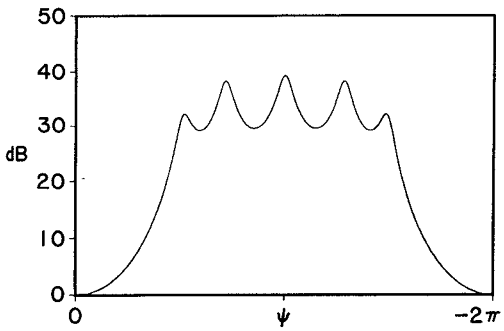

Fig. 7. Starting $D(\psi)$ function for six-hole directional coupler.

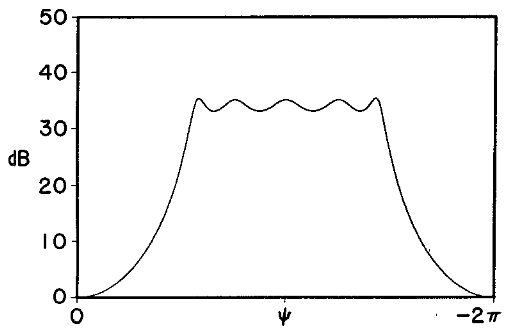

Fig. 8. Final rippled response for six-hole directional coupler.

which results, using (21), in the response shown in Fig. 7. For this example, the increments δa_0, δa_1, δa_2, δb_1, δb_2, and δK_1 need to be found from (23), using as driving function the values of $\delta(D-S)$ deduced from the ripple specification and the values of D at the three peaks and three dips in the right half of Fig. 7.

Five iterations yield the response shown in Fig. 8, with all ripple maxima and minima within 0.1 dB of what was specified. The final root positions are $w_0 = \exp(0.282 + j\pi)$, $w_1 = w_{-1}^* = \exp(0.247 + j2.35)$, and $w_2 = w_{-2}^* = \exp(0.108 + j1.77)$.

With the root positions known, (12) and (14) can be used to deduce the coupling coefficients. However, a study of (21) reveals that if a_0 is replaced by $-a_0$, or a_1 by $-a_1$, or a_2 by $-a_2$, the same $D(\psi)$ response occurs, the only difference being the value of K_1. More generally, if $N = 2M$ there are 2^M sets of roots that will produce the same response; if $N = 2M+1$ there are 2^{M+1} sets. Each of these sets gives a different solution for the scattering

TABLE II
MAGNITUDES OF THE SCATTERING COEFFICIENTS FOR THE SIX-HOLE DIRECTIONAL COUPLER WITH RIPPLED RESPONSE (FIG. 8)

	n	0	1	2	3	4	5
Solution No. 1	c_n	.0079	.0277	.0474	.0489	.0320	.0139
Solution No. 2	c_n	.0105	.0302	.0461	.0498	.0307	.0105
Solution No. 3	c_n	.0085	.0256	.0459	.0490	.0364	.0131
Solution No. 4	c_n	.0064	.0229	.0427	.0503	.0382	.0173

coefficients, and one is free to pick that solution which is easiest to realize physically. Further study shows that these solutions occur in pairs that are mirror symmetric versions of each other. In the present application there are thus four distinct sets of scattering characteristics to consider. With $C = 15$ dB these solutions are listed in Table II.

One can observe from the entries in this table that all four solutions lead to sets of hole sizes which lack symmetry, unlike the cases cataloged in Table I. One should choose the set with the least variation in nearest neighbors, since the first-order theory being used here is best justified when that occurs. Solution no. 2 seems best under that criterion but all four solutions are physically realizable.

By comparing the bandwidth exhibited in Figs. 6 and 8 one can determine the price paid for raising $D_{\min}$ and choosing a rippled response instead of one containing poles. In this case there was an improvement in $D_{\min}$ of 3 dB coupled with a 14% decrease in bandwidth. The iterative procedure can demonstrate larger improvements in $D_{\min}$ and/or decreases in the ripple magnitude, but at the cost of a lessened bandwidth. Trade-offs can be studied easily by inputting a variety of specifications to the computer program.

VI. CONCLUSIONS

A rapidly converging iterative technique has been demonstrated which will provide the design of multihole directional couplers when the directivity versus frequency is to be a modification of Chebyshev (all inverted lobes in the passband *not* at the same height). A *rippled* response can be also achieved where $D_{\min}$ is higher than Chebyshev at a modest reduction in bandwidth. The solutions are physically realizable.

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Robert S. Elliott (S'46–A'52–SM'54–F'61–LF'87) holds an A.B. in English literature and a B.S. in electrical engineering from Columbia University (1942, 1943), an M.S. and a Ph.D. in electrical engineering from the University of Illinois (1947, 1952), and an M.A. in economics from the University of California at Santa Barbara (1971).

He has been a Professor of Electrical Engineering at UCLA since 1957. His prior experience includes periods at the Applied Physics Laboratory of John Hopkins and at the Hughes Research Laboratories, where he headed the antenna research activities. He has also been on the faculty of the University of Illinois and was a founder of Rantec Corporation, serving as its first Vice President and Technical Director. He has recently completed a second two-year stint as a Distinguished Lecturer for the IEEE and was Chairman of the Coordinating Committee for the 1981 IEEE Symposium, held in Los Angeles.

Dr. Elliott is a member of the National Academy of Engineering, Sigma Xi, Tau Beta Pi, and the New York Academy of Sciences. He has received numerous teaching prizes, is the recipient of two Best Paper awards from the IEEE, and was honored with last year's AP-S Career Achievement Award.

Yong Uk Kim (S'82–M'87) received the B.S. degree in engineering science from the University of California, Berkeley, in 1980 and the M.S. and Ph.D. degrees in 1983 and 1987, respectively, in electrical engineering from UCLA.

His industrial experience includes periods at Lockheed and Hughes. In 1988 he joined TRW, Redondo Beach, CA, where he has been involved in spacecraft antenna systems.